Exercise 18

For the following exercises, find the domain, range, and all zeros/intercepts, if any, of the functions.

$$f(x) = \frac{1}{\sqrt{x-9}}$$

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Solution

$f(x)$ has a square root function, so the domain would be the set of all x where the argument is nonnegative. But since the square root is in the denominator, it also can't be zero.

$$x - 9 \geq 0 \quad \text{and} \quad x - 9 \neq 0$$

$$x \geq 9 \quad \text{and} \quad x \neq 9$$

Therefore, the domain is $\{x \mid x > 9\}$. There's a vertical asymptote at $x = 9$. $f(x)$ is continuous for $x > 9$, so it takes on all values between

$$f(9.0001) = \frac{1}{\sqrt{9.0001 - 9}} \approx 100$$

$$f(100\,000) = \frac{1}{\sqrt{100\,000 - 9}} \approx 0.003.$$

Putting in values of x even closer to 9 yields even higher values, and putting in values of x larger than $100\,000$ yields a number even closer to zero. Therefore, the range is $\{y \mid 0 < y < \infty\}$. Below is a graph of $f(x)$ versus x to confirm these results.